Quaternion to Direction Cosine Matrix Conversion
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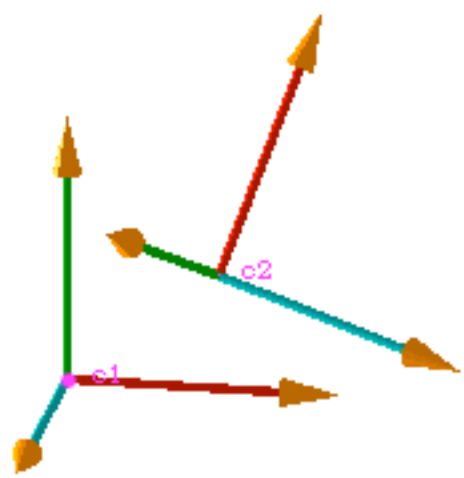
The columns of a direction cosine matrix are the components of unit vectors along the axes of an othogonal coordinate that has been rotated by the rotation which it describes. Therefore, constructing the direction cosine matrix corresponding to a quaternion is accomplished by rotating each of the orthogonal unit vectors by the quaternion and placing them in the appropriate columns of the matrix.

The rotation operation is:

$$V_{rot} = Q\,\boldsymbol{R}\,V = Q \otimes V \otimes Q^{*} \qquad \text{eqn 15}$$

(All references are to my paper**, Quaternion to Euler Angle Conversion for Arbitrary Rotation Sequence Using Geometric Methods**)

Example:



c2 is a coordinate frame rotated by the quaternion

$$Q_G = 0.360423 \;\; 0.439679 \;\; 0.391904 \;\; 0.723317 \qquad \text{eqn 17}$$

The three vectors to be rotated are:

v_1 = 1 0 0
v_2 = 0 1 0
v_3 = 0 0 1

After rotation, the three vectors are:

v_{1R} =	0.306185853	0.8838825	-0.35355216
v_{2R} =	-0.250000803	0.433011621	0.866024084
v_{3R} =	0.918557021	-0.176776249	0.353553866

Each of these vectors becomes a column of the direction cosine matrix:

$$DCM = \begin{matrix} 0.306185853 & -0.250000803 & 0.918557021 \\ 0.8838825 & 0.433011621 & -0.176776249 \\ -0.35355216 & 0.866024084 & 0.353553866 \end{matrix}$$